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Engineering Notes

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Maximization of Thrust–Torque Ratio of a Coaxial Rotor

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Introduction

In previous studies[1] on coaxial rotors, the rotational velocity was the same for both the upper and lower rotors. Here, the thrust–torque ratio, C_T/C_Q, is maximized without this rotational velocity condition, that is, rotational velocity of the upper rotor being the same as that of the lower rotor, and the effect of this rotational velocity condition on this maximization is clarified.

Materials and Methods

Table 1 shows the dimensions of the coaxial rotor used here. The upper and lower rotors both had two blades, and all four blades had the same shape. Two motors were used to rotate the rotors, one motor for each rotor (FK-180SH by Mabuchi Motor Corporation). The coaxial rotor and motors were mounted on a load cell (MMS-2402 by Nissho Electric Corporation), which measured both the thrust T generated by the coaxial rotor and the difference in torque $Q_d (=Q_u - Q_l)$ between the upper and lower rotors. The Q_u and Q_l were measured as follows. The coaxial rotor was rotated at a gear ratio of 8.6. The relation between the rotational velocity of the motor Ω_m and that of the coaxial rotor Ω, and the relation between the torque generated by the motor Q_m and that by the coaxial rotor Q are expressed by

$$\Omega = (1/8.6)\Omega_m, \qquad Q = 8.6 Q_m \tag{1}$$

The characteristics of an FK-180SH motor are expressed in an equation provided by the manufacturer:

$$Q_m(\mathrm{N\cdot m}) = -1.2\times 10^{-5}(\mathrm{N\cdot m\cdot s})\Omega_m + 5.4\times 10^{-3}(\mathrm{N\cdot m/V})V \tag{2}$$

where V (volts) is the voltage applied to the motor. The relationship among Q_m, V, and Ω_m for a dc motor is generally given by $Q_m = \alpha\Omega_m + \beta V$. From Eqs. (1) and (2), Q can be expressed by using Ω (radians per second) and V (volts) as

$$Q(\mathrm{N\cdot m}) = -8.7\times 10^{-4}(\mathrm{N\cdot m\cdot s})\Omega + 4.65\times 10^{-2}(\mathrm{N\cdot m/V})V \tag{3}$$

Q_u and Q_l can be estimated by measuring Ω and V of each rotor. The error in Eq. (3) will be discussed in the section, "Error and Uncertainty."

In the measurements of T, Q_d, Q_u, and Q_l, the pitch angles of the upper and lower rotors, θ_u and θ_l, which are measured at the blade root, were set to specific values. The rotational velocities of the upper and lower rotors, Ω_u and Ω_l, were adjusted to satisfy the conditions $T = 1.65$ N and $Q_d = 0$. In reality, the measurements were made under the following conditions:

$$|T - 1.65\ \mathrm{N}|/1.65\ \mathrm{N} < 0.1, \qquad |Q_d|/(Q_u + Q_l) < 0.03 \tag{4}$$

These errors in the measured values were so small that they were not expected to affect the conclusion under conditions where T is fixed and $Q_u = Q_l$, which is the second condition discussed in the "Results and Discussion" section.

For comparison, the characteristics of a single rotor were also measured. The rotational velocity of a single rotor was $\Omega = 50\pi$ (rad/s) and the Reynolds number Re defined at 75% spanwise position was 2.5×10^4. For the coaxial rotor, $40\pi < \Omega < 80\pi$ (rad/s), and $2\times 10^4 < Re < 4\times 10^4$. In the comparison, the difference in Reynolds number Re between the single and coaxial rotors can be ignored because the characteristics of the single rotor remain constant for $2\times 10^4 < Re < 4\times 10^4$.

C_T, C_Q, and the solidity σ are defined as

$$C_T = \begin{cases} T/\rho\pi R^2(\Omega R)^2 & \text{for a single rotor} \\ T/\rho\pi R^2[\{(\Omega_u + \Omega_l)/2\}R]^2 & \text{for a coaxial rotor} \end{cases} \tag{5}$$

$$C_Q = \begin{cases} Q/\rho\pi R^3(\Omega R)^2 & \text{for a single rotor} \\ (Q_u + Q_l)/\rho\pi R^3[\{(\Omega_u + \Omega_l)/2\}R]^2 & \text{for a coaxial rotor} \end{cases} \tag{6}$$

$$\sigma = \begin{cases} \sigma_S = 0.08 & \text{for a single rotor} \\ 2\sigma_S = 0.16 & \text{for a coaxial rotor} \end{cases} \tag{7}$$

Error and Uncertainty

The error in Q estimated by using Eq. (3) was determined as follows. Q acting on the single rotor was measured by the load cell for the various θ and Ω. This torque was also estimated by using Eq. (3). When $\Omega > 60\pi$ (rad/s), the Q measured based on Eq. (3) depended on individual motors of FK-180SH. When $\Omega < 60\pi$ (rad/s),

Table 1 Dimensions of coaxial rotor

Parameter	Value
Rotor radius, R	175 mm
Hinge offset	25 mm
Mean thickness ratio	8%
Position of maximum thickness	22% chordwise position from leading edge
Camber	8%
Position of maximum camber	30% chordwise position from leading edge
Nondimensionalized distance between the upper and lower rotors, d/R	0.29

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Q measured based on Eq. (3) was independent of individual motors. The measured Q agreed well with that measured using the load cell. Therefore, when $\Omega_u > 60\pi$, the measured Q_u can be replaced by $Q_u = Q_l + Q_d$, and when $\Omega_l > 60\pi$, the measured value of Q_l can be replaced by $Q_l = Q_u - Q_d$. The summation $Q_u + Q_l$ in Eq. (6) was then estimated by

$$Q_u + Q_l = \begin{cases} 2Q_l + Q_d, & \text{when} \quad \Omega_u > 60\pi\,(\text{rad/s}) \\ 2Q_u - Q_d, & \text{when} \quad \Omega_l > 60\pi\,(\text{rad/s}) \end{cases} \qquad (8)$$

Note that either Ω_u or Ω_l was less than 60π in the measurements done in this study.

Errors in the measured T and $Q_u + Q_l$ were estimated to be less than 5%, and those in the measured Ω_u and Ω_l were negligibly small. Therefore, the error in either C_T or C_Q was less than 5%. The error in (C_T/C_Q) was less than 10% (=5% + 5%), because (C_T/C_Q) is proportional to TR/Q as indicated by Eqs. (5) and (6). It will be shown later that the conclusion (i.e., the rotational velocity of the upper rotor is the same as that of the lower rotor, when the thrust–torque ratio of a coaxial rotor in hovering flight is maximum) is not significantly affected.

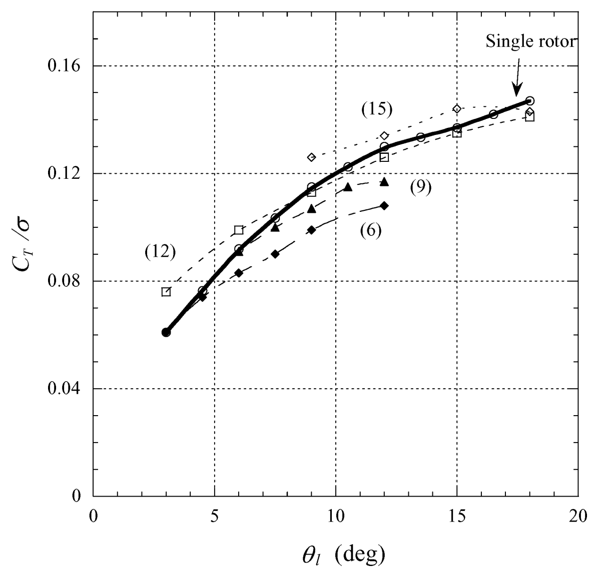

Fig. 1 Thrust coefficient C_T/σ for various upper-rotor pitch angle θ_u (parentheses) and lower-rotor pitch angle θ_l, where θ_u and θ_l are in degrees.

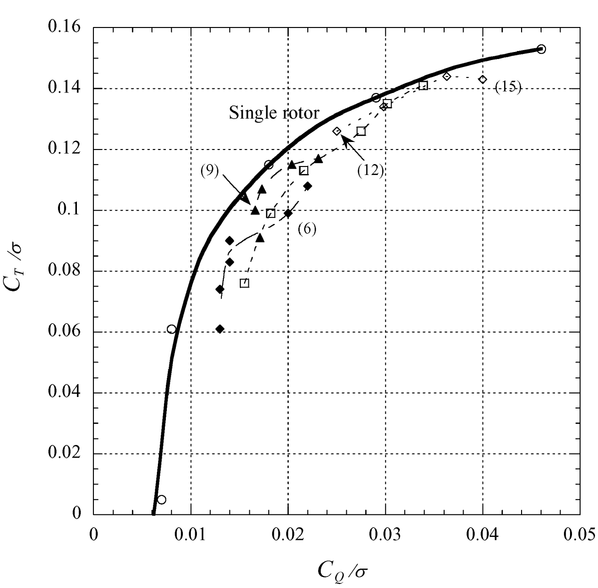

Fig. 2 Thrust coefficient C_T/σ vs torque coefficient C_Q/σ for various upper-rotor pitch angle θ_u (parentheses) and lower-rotor pitch angle θ_l, where θ_u and θ_l are in degrees.

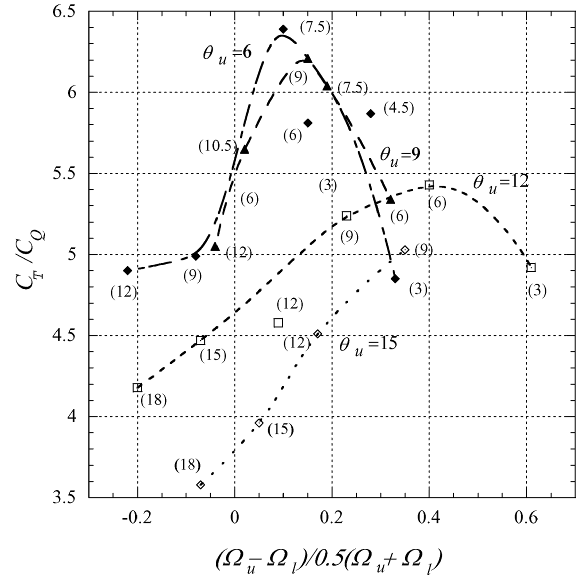

Fig. 3 Thrust–torque ratio C_T/C_Q vs ratio of the difference in rotor rotational velocity, $\Omega_u - \Omega_l$, to averaged rotor rotational velocity Ω_u and Ω_l, $(\Omega_l - \Omega_u)/0.5(\Omega_l + \Omega_u)$, for various upper-rotor pitch angle θ_u and lower-rotor pitch angle θ_l (parentheses), where θ_u and θ_l are in degrees.

Results and Discussion

Figures 1 and 2 show (θ_u, θ_l) vs C_T/σ and C_T/σ vs C_Q/σ, respectively, for the single and coaxial rotors for various upper-rotor pitch angle θ_u and lower-rotor pitch angle θ_l. The thrust–torque ratio C_T/C_Q for the single rotor was maximum (about 8) when $\theta \approx 5$ deg. In contrast, C_T/C_Q for the coaxial rotor was maximum (6.4) when $(\theta_u, \theta_l) = (6\text{ deg}, 7.5\text{ deg})$.

Figure 3 shows the ratio between $(\Omega_l - \Omega_u)/0.5(\Omega_l + \Omega_u)$ and C_T/C_Q for the coaxial rotor for various θ_u and θ_l. With decreasing θ_u, the maximum C_T/C_Q increased and $(\Omega_l - \Omega_u)/0.5(\Omega_l + \Omega_u)$ for the maximum C_T/C_Q decreased. The C_T/C_Q was maximum when $(\theta_u, \theta_l) = (6\text{ deg}, 7.5\text{ deg})$. At $(\theta_u, \theta_l) = (6\text{ deg}, 7.5\text{ deg})$, $\Omega_l - \Omega_u$ was near 0, namely, $\Omega_u \approx \Omega_l$. Therefore, C_T/C_Q of a coaxial rotor in hovering flight can be maximized by utilizing either of the following two conditions. Note that condition 1 was used previously[1,2] and that condition 2 was used in this study.

1) T is fixed, the condition $\Omega_u = \Omega_l$ is assumed, and the variables are $\Omega_u = \Omega_l$, θ_u, and θ_l. As a result, the condition $Q_u = Q_l$ is satisfied[2] for the maximization of C_T/C_Q.

2) T is fixed, the condition $Q_u = Q_l$ is assumed, and the variables are Ω_u, Ω_l, θ_u, and θ_l. As a result, the condition $\Omega_u = \Omega_l$ is satisfied for the maximization of C_T/C_Q.

This result means that either the condition $Q_u = Q_l$ or $\Omega_u = \Omega_l$ can be applied to maximize C_T/C_Q of a coaxial rotor in hovering flight.

Conclusions

The rotational velocity of the upper rotor is the same as that of the lower rotor, when the thrust–torque ratio of a coaxial rotor in hovering flight is maximum. Therefore, when maximizing C_T/C_Q of a coaxial rotor in hovering flight, the condition $\Omega_u = \Omega_l$ can be applied.

References

[1] Harrington, R. D., "Full-Scale-Tunnel Investigation of the Static-Thrust Performance of a Co-axial Helicopter Rotor," NACA TN 2318, 1951.

[2] Nagashima, T., Ohuchi, H., and Sasaki, F., "Optimum Performance and Load Sharing of Co-axial Rotor in Hover," *Journal of Japan Society for Aeronautical and Space Sciences*, Vol. 26, No. 293, 1978, pp. 325–333 (in Japanese).